Kindergarten Mathematics, Quarter 1, Unit 1.1

Building, Drawing, and Analyzing Shapes: Basic 2-D shapes; Matching, Identifying, Sorting, Building, Describing Shapes in their Environment

Overview

Number of instructional days: 5 (1 day = 45–60 minutes) 1st full week if school

Content to be learned

- Use spatial reasoning to build and model shapes in the world.
- Represent shapes in the world by drawing.
- Correctly name shapes regardless of their orientation or overall size.

Mathematical practices to be integrated

Model with mathematics.

- Analyze and reflect on their shape models and drawings.

Use appropriate tools strategically.

- Use components (e.g., sticks and clay balls) to build shapes.
- Use examples (if needed) to draw shapes.

Essential questions

- How would you build (this modeled) shape?
- How would you draw (this modeled) shape?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**K.G**

Analyze, compare, create, and compose shapes.

K.G.2 Correctly name shapes regardless of their orientation or overall size

K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

Many students have learned about shapes in their environment. Some students have learned names of some shapes.

Current Learning

Modeling shapes by building shapes from components (e.g., sticks and clay balls) and drawing shapes is at the developmental level. The students expand their physical world using geometric ideas and vocabulary. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Future Learning

Students will use these skills to compose two-dimensional shapes or three-dimensional shapes to create a composite shape, and compose new shapes from the composite shape. They will build and draw shapes to possess defining attributes in 1st grade.

Additional Findings

According to *Principles and Standards for School Mathematics* "Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and can be important tools in problem solving" (pg. 41).

Kindergarten Mathematics, Quarter 1, Unit 1.2

Developing an Understanding of Numbers 0-5 by Writing and Representing Numbers. Develop Rapid visual and Kinesthetic Recognition of Numbers to 5 on Fingers

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Represent the number counted with written numerals 0-5.
- Count to answer "how many?" questions about 0-5 things arranged in a line.
- Develop one-to-one correspondence

Mathematical practices to be integrated

Use appropriate tools strategically.

- Count objects using a variety of manipulatives arranged in a line.
- Write numerals 0-5 using a number chart/grid if needed

Attend to precision

- Answer "how many?" questions about 0-5 things with teachers and peers
- Communicate the action of counting to others.

Evidence Statements:

i) Tasks may have content

ii) Tasks should include a range of counting exercises to exercises to answer: how many" objects indifferent arrangements progressing to the more difficult action of counting out a given number of objects.

iii) Interviews (individual or small group) should target students'' abilities to meet this evidence statement

Essential questions

- How can you find the number of objects? *(teachers of manipulatives)*
- How do you count to 5?
- How do you know how many objects are in this line/group?
- How many objects do you have? Show me how you know.
- How can you show the number of objects you counted? (numerals 0-5)

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality	K.CC

Know number names and the count sequence.

K.CC.3 Write numbers. Represent a number of objects with a written numeral (with 0 representing a count of no object)

Count to tell the numbers of objects.

K.CC.5 Count to answer "how many?" questions about things arranged in a line.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Many students have learned about numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

This unit is at the developmental level. Students count objects accurately and represent the number counted with a number. The teacher needs to provide developmental activities of their choice appropriate to the unit. The students expand on the understanding that numbers and counting are required in the world around them. Understanding numbers is a critical area of kindergarten instruction. Counting and Cardinality is only addressed in kindergarten in the CCSS.

Future Learning

Students will use counting skills and will extend knowledge of written numerals in future grades. In first grade, students will represent a number of objects with a written numeral to 120. They will apply their understanding of numbers in all future numerical and operational thinking.

Additional Findings

According to *Principles and Standards for School Mathematics,* "Through problem solving, students can explore and solidify their understandings of number. Young children's earliest mathematical reasoning is likely to be about number situations, and their first mathematical representations will probably be of numbers" (p 32).

Kindergarten Mathematics, Quarter 1, Unit 1.3

Developing Understanding of Numbers 0–10 and Writing and Representing Numbers 0-10

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Count objects in the standard order, 0–10
- Count pairing each object with one and only one number name (one-to-one correspondence).
- Understand that the last number said tells the number of objects counted.
- Understand that the successive number name refers to a quantity that is one larger.
- Represent the number counted with written numerals 0-10.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Use quantitative reasoning to understand that the successive number name refers to a quantity that is one larger.
- Count each object only one time.

Construct viable arguments and critique the reasoning of others.

- Count objects in standard order (0–10) with teacher and peers.
- Peer check the number of objects counted.

Evidence Statements:

i) Tasks may have content

ii) Tasks should include a range of counting exercises to exercises to answer: how many" objects indifferent arrangements progressing to the more difficult action of counting out a given number of objects.

iii) Interviews (individual or small group) should target students" abilities to meet this evidence statement

Essential questions

- How can you find the number of objects? *(teachers of manipulatives)*
- How do you count to 10?
- How do you know how many objects are in this line/group?
- How many objects do you have? Show me how you know.
- How can you show the number of objects you counted? (numerals 0-10)
- How many objects do we have if we count one more? (up to 10)

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality	**K.CC**

Know number names and the count sequence.

K.CC.3 Write numbers. Represent a number of objects with a written numeral (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name.

b. Understand that the last number name said tells the number of objects counted.

c. Understand that each successive number name refers to a quantity that is one larger.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings,

diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

This unit focuses on understanding the relationship between numbers and quantities, 0–10, and connecting counting to cardinality. This unit is at the developmental level. Students learn that the last number said is the amount counted. They count using one-to-one correspondence. Students recognize that each number counted is one more than the previous number. Understanding numbers is a critical area of kindergarten instruction. Counting and Cardinality is only addressed in kindergarten in the CCSS.

Future Learning

In first grade, students will extend counting to 120. They will count orally starting at any number less than 120. Students will relate counting and cardinality to addition strategies such as "counting on."

Additional Findings

According to *Principles and Standards for School Mathematics,* "They can associate number words with small collections of objects and gradually learn to count and keep track of objects in larger groups. They can establish one-to-one correspondence by moving, touching, or pointing to objects as they say the number words"(p. 79).

Kindergarten Mathematics, Quarter 1, Unit 1.4

Understanding and Describing Shapes and Space, Spatial Relationships and Structuring, and Orientation (i.e. above, below, next to, beside, etc.)

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Describe objects in the environment using names of shapes
- Describe relative positions of objects (above, below, beside).
- Identify and name squares, circles, triangles, rectangles and hexagons regardless of orientation and size.

Mathematical practices to be integrated

Look for and make structure

- Students recognize the structure of shapes (i.e. number of sides/lines, closed figure, straight lines).

Essential questions

- What are the attributes of a _____? (given a shape)
- What are some objects in the environment with a shape of a _____? Why do you think it has that shape?
- Where is the object compared to another object?
- Is this object (above, below, beside, etc.) the other object

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**K.G**

Identify and describe shapes (squares, circles, triangles, rectangles, rectangles, hexagons)

K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects in terms such as *above, below, beside, in front of, behind, and next to.*

K.G.2 Correctly name shapes regardless of their orientation or overall size.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

Students have identified and compared two-dimensional shapes. They have classified them into given categories.

Current Learning

Students are building on earlier understanding from previous units. Students describe 2-D objects in the environment using names of shapes and describe relative position of these objects using terms such as above, below, beside, in front of, behind, and next to. Students correctly name shapes regardless of their orientation or size.

Future Learning

First graders will distinguish between defining attributes and non-defining attributes. Students compose two-dimensional or three-dimensional shapes to create a composite shape.

Additional Findings

According to *Curriculum Focal Points,* children interpret the physical world with geometric ideas. They use basic shapes and spatial reasoning modeling objects in their environment and constructing more complex shapes.

Kindergarten Mathematics, Quarter 2, Unit 2.1

Classification and Sorting Objects into Categories; More or Less up to 10

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Use matching and counting strategies to compare two groups of objects.
- Verbally explain whether one group is greater than, less than, or equal to the number of objects in a second group.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.
- Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.

Model with mathematics.

- Students can apply the mathematics they know to solve problems in everyday life.
- Use manipulatives, drawings, equations, actions, and fingers to compare two quantities.

Essential questions

- Show me the group that is equal to this _____ (teacher's choice up to 10 objects).
- Show me a number that is greater than or less than _____ (a given number up to 10).
- How do you know this number is greater than, less than, or equal to _____ (any given number up to 10)?
- How do you know if two groups of objects are equal?
- Is your group of objects less than, greater than, or equal to my group of objects? How do you know?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality K.CC

Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]

[1] Include groups with up to ten objects.

Geometry K.G

Analyze, compare, create, and compose shapes.

K.G.2 Correctly name shapes regardless of their orientation or overall size.

Measurement and Data K.MD

K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]

[3] Limit category counts to be less than or equal to 10.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Some students started school with some knowledge and exposure to counting. Some students may have started school with some knowledge of shapes.

Current Learning

In quarter 1, students develop understanding of numbers up to 0. Students identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies for quantities up to 10 objects. In this unit, the teacher will be providing the categories for students to sort.

Future Learning

In quarter 3 students will be comparing written numerals to decide which is greater than, less than, or equal to. In first grade, students will compare two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, <.

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics*, "The second foundation of mathematical reasoning is language—symbols, terms, and other representations and their definitions" (p. 30).

Kindergarten Mathematics, Quarter 2, Unit 2.2

Developing an Understanding of Numbers 11-20; Writing and Representing Numbers 11-20

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Compose numbers from 11–19 into ten ones and some further ones.
- Decompose numbers from 11–19 into ten ones and some further ones.
- Record each composition or decomposition by making a drawing or writing an equation.
- Understand that these numbers (11–9) are composed of ten ones and 1–9 additional ones.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Monitor student thinking as they compose and decompose numbers between 11–19.
- Question students about patterns and properties of operation.

Construct viable arguments and critique the reasoning of others.

- Focus on correct, as well as incorrect, compositions and decompositions of numbers between 11–19.
- Provide time for communication and discussion of place value.

Evidence Statements

i) Tasks should focus on the understanding of numbers from 11 to 19 as composed of ten "ones" and some additional number of "ones".
ii) Tasks should require students to record their thinking with a drawing or equation.
iii) Interviews (individual or small group) should target this understanding of composing and decomposing the teen numbers into ten "ones" and some additional number of "ones".

Essential questions

- How can you put together this number into 10 ones and some more ones?
- How can you take apart this number into 10 ones and some more ones?
- How does your drawing represent the number ____ (11–19) using ten ones and some more ones?
- Given a group of ten, how many more ones do you need to show _____? How do you know? (numbers from 11 to 19)

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	**K.NBT**

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7X8 equals the remembered 7X5+7X3, in preparation for learning about the distributive property. In the expression x2+9x+14, older students can see the 14 as 2x7 and the 9 as 2+7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions as single objects or as being composed of several objects. For example they can see $5-3(x-y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real number x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y-2)/(x-1)=3. Noticing the regularity in the way terms cancel when expanding (x-1)(x+1), (x-1)(x2+x+1), and (x-1)(x3+x2+x+1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

This unit is at a developmental level. Students compose (put together) and decompose (take apart) numbers from 11–19 into ten ones and some further ones, e.g. by using objects or drawings. They record each composition or decomposition by drawing or equation (e.g., 18 = 10 + 8). Notice how the equal sign represents the situation or decomposition, not a calculation to an answer. Students understand that these numbers are composed of ten ones and 1–9 separate ones. This is the foundation to place value that will be addressed in future grades.

Kindergarten students are discrete thinkers. Therefore, models should represent 10 distinct ones and some more ones. Models— such as snap cubes—should break apart easily into 10 ones. Place-value rods representing 10 should not be used at this grade level.

Future Learning

In first grade, students will understand that the two digits of a two-digit number represent amounts of tens and ones.

Additional Findings

According to *Adding It Up*, "Research indicates that students' experiences using physical models to represent hundreds, tens, and ones can be effective if the materials help them think about how to combine quantities and eventually how these processes connect with written procedures. The models, however, are not automatically meaningful for students; the meaning must be constructed as they work with the materials" (p. 198).

Kindergarten Mathematics, Quarter 2, Unit 2.3

Counting and Writing to 20; Counting Forward to 20 From any Number

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Count forward beginning from a given number up to 20.
- Count within the known sequence up to 20.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.

Attend to Precision

- Students will communicate with precise language
- Students recognize general methods of counting.

Evidence Statements

i) Tasks may have a context.
ii) Tasks should include a range of counting exercises to answer "how many" objects in different arrangements progressing to the more difficult action of counting out a given number of objects.
iii) Interviews (individual or small group) should target students' abilities to meet this evidence statement.

Essential questions

- How do you count to 20?
- How do you count to 20 starting from the number _____?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality **K.CC**

Compare numbers.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1). [1]

1 Counting forward is a prerequisite to counting on. It is beginning with a given number within the known sequence; finally understand that each successive number name refers to a quantity that is one larger.

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 to 20 (with 0 representing a count of no objects).

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7X8 equals the remembered 7X5+7X3, in preparation for learning about the distributive property. In the expression x2+9x+14, older students can see the 14 as 2x7 and the 9 as 2+7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions as single objects or as being composed of several objects. For example they can see $5-3(x-y)_2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real number x and y.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

Counting forward to 20 from any number is introduced in this unit, and instruction should be at the developmental level. Counting forward to 20 from any number is a critical area of kindergarten instruction. Students will use the skill in future units to count within 100, and being able to count forward.

Counting from a given number is a foundational skill for developing understanding of addition and subtraction which will be addressed later this year.

It is important to continue to practicing and developing counting skills that were introduced in earlier units as students deepen their understanding of number in this unit

Future Learning

Students will use the skill of counting forward, starting at any number less than 120, in first grade.

Additional Findings

According to Principles and Standards for School Mathematics, “research has shown that counting is a foundation for students’ early work with numbers (e.g., NCTM [2005])” (p. 79). According to Principles and Standards for School Mathematics, “research has shown that all the mathematics proposed for prekindergarten through grade 12 is strongly grounded in number (e.g., NCTM [2005])” (p. 32).

Kindergarten Mathematics, Quarter 2, Unit 2.4

Addition Within 5 With Objects, Fingers, and Math Drawings. Utilizing Multiple Positions of Equations

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Represent addition situations with objects, fingers, mental images, drawings, sounds (e.g. claps), and acting out situations, verbal explanations, expressions, or equations.
- Solve addition word problems.
- Add within 5 using objects or pictures to represent the problem.
- Fluently add and subtract within 5.
- Focus on quantities and their relationships.
- Equations with the number on the left and the operation on the right e.g. 5=2+3; understanding that both sides have the same value.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Make sense of problems using a variety of methods, such as acting out, drawings, fingers, etc.
- Look for key vocabulary terms in word problems.
- Use concrete objects or pictures to help conceptualize and solve a problem.

Model with mathematics.

- Apply the mathematics they know to solve problems in everyday life.
- Represent story problems using objects, pictures, equations, or expressions.

Evidence Statements

i) Tasks should provide students opportunities to demonstrate fluency for addition within 5 and to apply different solution methods.

ii) Interviews (individually or small groups) should target students' abilities to meet this evidence statement.

Essential questions

- How did you solve this problem?
- How do your pictures, equations, and/or objects represent this problem?
- How can you model this addition problem by using objects or by drawing a picture?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Focus on quantities and their relationships. Can be acted out or presented with pictures or words.

Drawings facilitate reflection and discussion.

K.OA.5 Fluently add and subtract within 5.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7X8 equals the remembered 7X5+7X3, in preparation for learning about the distributive property. In the expression x2+9x+14, older students can see the 14 as 2x7 and the 9 as 2+7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions as single objects or as being composed of several objects. For example they can see $5-3(x-y)_2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real number x and y.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

This unit is at a developmental level. Students add within 5 working towards fluency, using both word problems and written equations. Students work to understand that both sides of the equal sign are the same. Students need to begin making equations where one number is on the left and an operation is on the right (e.g. 5=2+3 & 2+3=5). Note that the CCSSM introduction on page 9 states, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but not required."

Future Learning

In first grade, students will solve addition and subtraction problems within 20, with fluency to 10.

Additional Findings

According to *Adding It Up*, "Word problems are often thought to be more difficult than simple number sentences or equations. Young children, however, find them easier. If the problems pose simple relationships and are phrased clearly, preschool and kindergarten children can solve word problems involving addition, subtraction, multiplication, and division." (p. 169).

Kindergarten Mathematics, Quarter 3

Developing Understanding of Numbers 0-50 and Introducing Counting by 10's to 100 (Throughout Quarter 3)

Overview

Number of instructional days: Daily (1 day = 45–60 minutes)

Content to be learned

- Count forward beginning from a given number up to 50.
- Count within the known sequence up to 50.
- Count by tens up to 100.

Mathematical practices to be integrated

Attend to precision.

- Communicate with precise language.

Look for and express regularity in repeated reasoning.

- Recognize general methods of counting.

Essential questions

- How do you count to 50?
- How do you count to 50 by tens?
- How do you count to 50 from the number _____?
- How is counting to 50 the same or different than counting to 20?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality K.CC

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

Say counting words first then use these numbers to count objects or to tell the number of objects. Pair each word with one object. Facilitate by pointing to objects or moving them. Count object in a line, later count in arrangements. Lastly count out a given number of objects.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y-2)/(x-1)=3. Noticing the regularity in the way terms cancel when expanding (x-1)(x+1), (x-1)(x2+x+1), and (x-1)(x3+x2+x+1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

Students are in the developmental level during this unit. It is a critical area and requires more emphasis. Students count to 50 by 1s and 10s. In previous units, students counted by ones to 5, 10, and 20. By the end of the year, students count to 100 by ones and tens.

Future Learning

In first grade, students will extend the counting sequence by counting to 120, starting at any number less than 120.

Additional Findings

According *Principles and Standards or School Mathematics*, "Representing numbers with various physical materials should be major part of mathematics instruction in the elementary grades." (p. 33).

Kindergarten Mathematics, Quarter 3, Unit 3.1

Introduce Subtraction Within 5 With Objects, Their Fingers and Math Drawings

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Represent subtraction situations with objects, fingers, mental images, drawings, sounds (e.g. claps), acting out situations, verbal explanations, expressions, or equations.
- Develop conceptual understanding of subtraction as "take from" and "take apart".
- Solve subtraction word problems.
- Add and subtract within 5 using objects or pictures to represent the problem.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use concrete objects or pictures to help conceptualize and solve a problem.

Model with mathematics.

- Apply the mathematics they know to solve problems in everyday life.
- Identify important quantities in practical situations and map their relationships using tools and diagrams (e.g., ten-frame)

Evidence Statement:

i) Tasks should provide students opportunities to demonstrate fluency for addition and subtraction within 5 and to apply different solution methods.

ii) Interviews (individually or small group) should target students' abilities to meet this evidence statement.

Essential questions

- How did you solve this problem?
- How do your pictures, equations, and/or objects represent this problem?
- How can you model this subtraction problem by using objects or by drawing a picture?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

K.OA.5 Subtract within 5.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships

mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

This unit is at a developmental level. Students add and subtract within 5 in previous units of study. Students now add within 10 and subtract within 5, with fluency to 5, using both word problems and written equations. Students decompose numbers less than or equal to 10 into two different equations (can be shown by drawings or using objects) and record the equations (or drawings). Note that the CCSSM introduction on page 9 states, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but not required." Students find the number that makes 10 when added to any given number from 1–9.

Future Learning

In first grade, students will solve addition and subtraction problems within 20, with fluency to 10.

Additional Findings

According to *Adding It Up*, "Word problems are often thought to be more difficult than simple number sentences or equations. Young children, however, find them easier. If the problems pose simple relationships and are phrased clearly, preschool and kindergarten children can solve word problems involving addition, subtraction, multiplication, and division." (p. 169).

Kindergarten Mathematics, Quarter 3, Unit 3.2

Adding and Subtracting up to 5

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Represent addition and subtraction situations with objects, fingers, mental images, drawings, sounds (e.g. claps), acting out situations, verbal explanations, expressions, or equations.
- Solve addition and subtraction word problems.
- Add and subtract within 5 using objects or pictures to represent the problem.
- Fluently add and subtract within 5

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Make sense of problems using a variety of methods, such as acting out, drawings, fingers, etc.
- Look for key vocabulary terms in word problems.
- Use concrete objects or pictures to help conceptualize and solve a problem.

Model with mathematics.

- Apply the mathematics they know to solve problems in everyday life.
- Represent story problems using objects, pictures, equations, or expressions.

Evidence Statements:

i) Tasks should include the following problem situations: "Add To" and "Take From" – Result Unknown Problems, and "Put Together/Take Apart" – Total Unknown and Both Addends Unknown Problems (for more information see CCSS Table 1, p. 88 and OA Progression, p. 9)

ii) Interviews (individual or small group) are used to assess mastery of different problem types.

Essential questions

- How did you solve this problem?
- How do your pictures, equations, and/or objects represent this problem?
- How can you model this addition problem by using objects or by drawing a picture?
- How can you model this subtraction problem by using objects or by drawing a picture?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**K.OA**

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 5, e.g., by using objects or drawings to represent the problem.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of

the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

This unit is at a developmental level. Students add and subtract within 5 in previous units of study. Students now add and subtract within 5, with fluency to 5, using both word problems and written equations. Students decompose numbers less than or equal to 10 into two different equations (can be shown by drawings or using objects) and record the equations (or drawings). Note that the CCSSM introduction on page 9 states, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but not required." Students find the number that makes 10 when added to any given number from 1–9.

Future Learning

In first grade, students will solve addition and subtraction problems within 20, with fluency to 10.

Additional Findings

According to *Adding It Up*, "Word problems are often thought to be more difficult than simple number sentences or equations. Young children, however, find them easier. If the problems pose simple relationships and are phrased clearly, preschool and kindergarten children can solve word problems involving addition, subtraction, multiplication, and division." (p. 169).

Kindergarten Mathematics, Quarter 3, Unit 3.3

Measurement: Compare length (align end points) and Weight. Describe and Compare Measureable Attributes. Represent Measurement Data on Picture/Bar Graph

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Describe measureable attributes of objects.
- Directly compare two objects with a measureable attribute in common.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Justify conclusions and communicate them to others.

Essential questions

- How can you compare these objects? What can you measure?
- Which objects weighs more/less? How do you know?
- Which object is longer/shorter (taller/shorter, wider/thinner, heavier/lighter, etc.)? How do you know?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	**K.MD.**

Describe and Compare Measurable Attributes

K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/ "less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7X8 equals the remembered 7X5+7X3, in preparation for learning about the distributive property. In the expression x2+9x+14, older students can see the 14 as 2x7 and the 9 as 2+7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions as single objects or as being composed of several objects. For example they can see $5-3(x-y)_2$ as 5 minus a positive number times a square and use that to realize that its value cannot

be more than 5 for any real number x and y.

Clarifying the Standards

Prior Learning

Some students may be familiar with the terms longer/shorter, taller/shorter, and heavier/lighter.

Current Learning

In this unit of study, students are introduced to measuring and comparing objects looking specifically at lengths and weight. Teacher needs to discuss explicitly the importance of aligning endpoints when measuring. Area and volume are not instructional focus for kindergarten.

Future Learning

In first grade students will be ordering three objects by length and comparing the length of two objects indirectly by the third object.

Kindergarten Mathematics, Quarter 3, Unit 3.4

Matching, Identifying, Analyzing, Sorting, Describing and Composing 3-D Shapes

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Identify shapes as three-dimensional.
- Analyze and compare three-dimensional shapes, in different sizes and orientations.
- Use informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes.
- Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Justify conclusions, communicate them to others, and respond to the arguments of others.

Look for and make use of structure.

- Sort a collection of shapes according to how many sides the shapes have.
- Recognize that shapes can be identified by attributes such as number of sides, corners, flat, or solid.

Essential questions

- What makes this is a three-dimensional shape?
- How are these shapes the same (give a student several shapes to compare)?
- How are these shapes different (give a student several shapes to compare)?
- How can you use different shapes to make a new shape?
- What shapes can you make given two triangles, two rectangles, or two squares?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**K.G**

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

K.G.6 Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?

Perceive a variety of shapes in their environment, describe shapes in own words recognize and informally name 3 dimensional shapes e.g., balls, boxes, cans; in context of solving problems in block building drawing pictures and simple maps.

Name and describe 3 dimensional shapes with math vocabulary (sphere, cube, cylinder, and cone. ID faces of 3D shapes as 2D geometric figures. ID as 2D (flat) or 3D (solid).

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students have seen shapes in their environment.

Current Learning

Students identify shapes as either two- or three-dimensional by describing two-dimensional figures as "flat," and three-dimensional figures as "solid." Students analyze and compare two- and three-dimensional shapes using informal language to describe similarities and differences. Students compose simple shapes to form larger shapes.

Future Learning

In first grade, students will distinguish between defining and non-defining attributes. They will also compose two- or three-dimensional shapes to create a composite shape.

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics*, "Children's ideas about shape do not come from passive looking. Instead, they come as children's bodies, hands, eyes, and minds engaged in active construction. In addition, children need to explore extensively to fully understand them; merely seeing and naming pictures is insufficient" (p. 152).

Kindergarten Mathematics, Quarter 4

Developing Understanding of Numbers 0-100 and Introducing Counting by 10's to 100 (Throughout Quarter 4)

Overview

Number of instructional days: Daily (1 day = 45–60 minutes)

Content to be learned

- Count forward beginning from a given number up to 100.
- Count within the known sequence up to 100.
- Count by tens up to 100.

Mathematical practices to be integrated

Attend to precision.

- Communicate with precise language.

Look for and express regularity in repeated reasoning.

- Recognize general methods of counting.

Essential questions

- How do you count to 100?
- How do you count to 100 by tens?
- How do you count to 100 from the number _____?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality **K.CC**

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

Say counting words first then use these numbers to count objects or to tell the number of objects. Pair each word with one object. Facilitate by pointing to objects or moving them. Count object in a line, later count in arrangements. Lastly count out a given number of objects.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y-2)/(x-1)=3. Noticing the regularity in the way terms cancel when expanding (x-1)(x+1), (x-1)(x2+x+1), and (x-1)(x3+x2+x+1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

Students are in the developmental level during this unit. It is a critical area and requires more emphasis. Students count to 50 by 1s and 10s. In previous units, students counted by ones to 5, 10, and 20. By the end of the year, students count to 100 by ones and tens.

Future Learning

In first grade, students will extend the counting sequence by counting to 120, starting at any number less than 120.

Additional Findings

According *Principles and Standards or School Mathematics*, "Representing numbers with various physical materials should be major part of mathematics instruction in the elementary grades." (p. 33).

Kindergarten Mathematics, Quarter 4, Unit 4.2

Adding and Subtracting up to 10

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Represent addition and subtraction situations with objects, fingers, mental images, drawings, sounds (e.g. claps), acting out situations, verbal explanations, expressions, or equations.
- Solve addition and subtraction word problems.
- Add and subtract within 10 using objects or pictures to represent the problem.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Make sense of problems using a variety of methods, such as acting out, drawings, fingers, etc.
- Look for key vocabulary terms in word problems.
- Use concrete objects or pictures to help conceptualize and solve a problem.

Model with mathematics.

- Apply the mathematics they know to solve problems in everyday life.
- Represent story problems using objects, pictures, equations, or expressions.

Evidence Statements:

i) Tasks should include the following problem situations: "Add To" and "Take From" – Result Unknown Problems, and "Put Together/Take Apart" – Total Unknown and Both Addends Unknown Problems (for more information see CCSS Table 1, p. 88 and OA Progression, p.9)

ii) Interviews (individual or small group) are used to assess mastery of different problem types.

Evidence Statements:

i) Tasks may have a context.

ii) Tasks should focus on students' understanding of making 10 and representing their thinking.

iii) Interviews (individual or small group) should target students' abilities to meet this evidence statement.

Essential questions

- How did you solve this problem?
- How do your pictures, equations, and/or objects represent this problem?
- How can you model this addition problem by
- How can you model this subtraction problem by using objects or by drawing a picture?

using objects or by drawing a picture?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**K.OA**

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions

and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

This unit is at a developmental level. Students add and subtract within 5 in previous units of study. Students now add and subtract within 10, with fluency to 5, using both word problems and written equations. Students decompose numbers less than or equal to 10 into two different equations (can be shown by drawings or using objects) and record the equations (or drawings). Note that the CCSSM introduction on page 9 states, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but not required." Students find the number that makes 10 when added to any given number from 1–9.

Future Learning

In first grade, students will solve addition and subtraction problems within 20, with fluency to 10.

Additional Findings

According to *Adding It Up*, "Word problems are often thought to be more difficult than simple number sentences or equations. Young children, however, find them easier. If the problems pose simple relationships and are phrased clearly, preschool and kindergarten children can solve word problems involving addition, subtraction, multiplication, and division." (p. 169).

Kindergarten Mathematics, Quarter 4, Unit 4.3

Developing Foundations for Place Value 11–19

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Compose numbers from 11–19 into ten ones and some further ones.
- Decompose numbers from 11–19 into ten ones and some further ones.
- Record each composition or decomposition by making a drawing or writing an equation.
- Understand that these numbers (11–9) are composed of ten ones and 1–9 additional ones.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Monitor student thinking as they compose and decompose numbers between 11–19.
- Question students about patterns and properties of operation.

Construct viable arguments and critique the reasoning of others.

- Focus on correct, as well as incorrect, compositions and decompositions of numbers between 11–19.
- Provide time for communication and discussion of place value.

Evidence Statements:

i) Tasks should focus on the understanding of numbers from 11 to 19 as composed of ten "ones and some additional number of "ones."

ii) Tasks should require students to record their thinking with a drawing or equation.

iii) Interviews (individual or small group) should target this understanding of composing and decomposing the teen numbers into ten "one" and some additional number of "ones".

Essential questions

- How can you put together this number into 10 ones and some more ones?
- How can you take apart this number into 10 ones and some more ones?
- How does your drawing represent the number ____ (11–19) using ten ones and some more ones?
- Given a group of ten, how many more ones do you need to show _____? How do you know? (numbers from 11 to 19)

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten **K.NBT**

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Composing and decomposing numbers from 11-19 into ten ones and some further ones, builds from all this work, vital first step toward understanding base-ten notation for numbers greater than 9.

Work with numbers 11-19 to gain foundation for place value. Use objects, math drawings and equations to describe, explore and explain how the "teen numbers" are ten ones and some more ones. Count out a given teen number of objects, e.g. 12. Special attention to 11, 12, 13 to understand them due to English language. E.g. layered place value cards help see the 0 "hiding" under the ones place and the 1 in the tens place really is 10 (ten ones).

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7X8 equals the remembered 7X5+7X3, in preparation for learning about the distributive property. In the expression x2+9x+14, older students can see the 14 as 2x7 and the 9 as 2+7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions as single objects or as being composed of several objects. For example they can see $5-3(x-y)_2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real number x and y.

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y-

2)/(x-1)=3. Noticing the regularity in the way terms cancel when expanding (x-1)(x+1), (x-1)(x2+x+1), and (x-1)(x3+x2+x+1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

This unit is at a developmental level. Students compose (put together) and decompose (take apart) numbers from 11–19 into ten ones and some further ones, e.g. by using objects or drawings. They record each composition or decomposition by drawing or equation (e.g., 18 = 10 + 8). Notice how the equal sign represents the situation or decomposition, not a calculation to an answer. Students understand that these numbers are composed of ten ones and 1–9 separate ones. This is the foundation to place value that will be addressed in future grades.

Kindergarten students are discrete thinkers. Therefore, models should represent 10 distinct ones and some more ones. Models— such as snap cubes—should break apart easily into 10 ones. Place-value rods representing 10 should not be used at this grade level.

Future Learning

In first grade, students will understand that the two digits of a two-digit number represent amounts of tens and ones.

Additional Findings

According to *Adding It Up*, "Research indicates that students' experiences using physical models to represent hundreds, tens, and ones can be effective if the materials help them think about how to combine quantities and eventually how these processes connect with written procedures. The models, however, are not automatically meaningful for students; the meaning must be constructed as they work with the materials" (p. 198).

Kindergarten Mathematics, Quarter 4, Unit 4.1

Operations and Algebraic Thinking: Decompose Numbers Less than or equal to 10. Develop Equations in more than one way

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Decompose numbers less than or equal to 10 into pairs in more than one way.
- For any number from 1–9, find the number that makes 10 when added to the given number.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Make sense of problems using a variety of methods, such as acting out, drawings, fingers, etc.
- Look for key vocabulary terms in word problems.
- Use concrete objects or pictures to help conceptualize and solve a problem.

Model with mathematics.

- Apply the mathematics they know to solve problems in everyday life.

Evidence Statements

i) Tasks may have a context.
ii) Tasks should include a range of activities that focus on decomposing numbers less than or equal to 10 into pairs in more than one way.
iii) Tasks require students to record their thinking with a drawing or equation.
iv) Tasks should focus on students' understanding of making 10 and representing their thinking.
v) Interviews (individual or small group) should target students' abilities to meet this evidence statement.

Essential questions

- How did you solve this problem?
- How do your pictures, equations, and/or objects represent this problem?
- Can you show me more than one way to break apart this number (equal to or less than 10)? How would you record this?
- What number can you add to _____ to make the equation equal to 10?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Clarifying the Standards

Prior Learning

Many students have learned numbers and have been exposed to using them in their environment. Some students have already learned to count and represent numbers.

Current Learning

This unit is at a developmental level. Students decompose numbers less than or equal to 10 into two different equations (can be shown by drawings or using objects) and record the equations (or drawings). Students find the number that makes 10 when added to any given number from 1–9. Students need to continue to understand that equations are equal on both sides of the equal sign (e.g. 2+3=5 is the same as 5=2+3).

Future Learning

In first grade, students will compose and decompose numbers to solve addition and subtraction problems where they have to find an unknown.